VALUING THE UNEXPIRED RISK OF MOTOR SERVICE PLANS

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ABSTRACT

A motor service plan transfers cost of scheduled future services from a customer to the issuer. Automobile manufacturers often specify when a vehicle is due for service in terms of whichever occurs first between (1) travelling a specified distance; and (2) time from delivery date or previous service. This two-dimensional specification leads to difficulty in valuing motor service plans by causing uncertainty in (1) the timing of future scheduled services; and (2) the count of future services. This paper proposes a discounted cash-flow framework to value the unexpired risk on a motor service plan portfolio. A survival model is employed to project both the timing and count of future services. The survival model is also utilised to estimate the probability that a scheduled service is claimed. Simulation is used to show how the value of unexpired risk is distributed. The proposed valuation method is demonstrated using portfolio data from a provider of passenger vehicle motor service plans. The results indicate that higher vehicle usage rates increase the value of unexpired risk. The ensuing findings demonstrate the importance of considering vehicle usage distribution when assessing unexpired risk of motor service plans.

KEYWORDS

Motor service plans; extended service plan; survival model; simulation

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1. INTRODUCTION AND BACKGROUND

It is standard practice that new vehicles are sold with a service plan. This means that a new vehicle's sale price includes the cost of scheduled services meant to ensure that the vehicle remains in good working condition. Vehicle manufacturers often specify cover in terms of maximum odometer reading or time, whichever occurs first. For example, a service plan quoted as providing cover of 'five years/100,000 kilometres' means that the manufacturer stops meeting costs of scheduled services if the vehicle's odometer reading reaches 100,000 kilometres or when the vehicle is aged five years, whichever occurs first. Likewise, manufacturers often specify when a vehicle is due for scheduled service as the earlier of either (1) travelling a specified distance; or (2) time from delivery date or previous service. For example, a vehicle can be due for service after travelling 20,000 kilometres or after one year from previous service.

Vehicle manufacturers also tend to offer extended service plans, which provide cover after expiry of the base service plan (Wang, 2010). Customers have an option on whether to purchase additional cover provided by an extended service plan, unlike a base service plan that is always tied to the sale of a new vehicle (Heese, 2012). Over the past decade, the global market in extended motor service plans has grown considerably (Wang, 2010). This is partly because of an increase in the number of third parties providing these contracts. Such third parties include insurance companies, banks, financial services divisions of automobile manufacturers and organisations with distribution rights of a motor vehicle. Most of these third parties sell their extended service plans through vehicle dealerships.

Although closely related, motor service plans are different from motor warranties. Motor servicing entails replacing 'service' items, meaning components that a manufacturer specifies as requiring routine replacement. What seems to vary across providers, at least in the South African market, is the extent to which 'wear-and-tear' items are included in motor service plans. Unlike 'service' items, 'wear-and-tear' items are replaced on a scheduled service only if deemed necessary at the time the vehicle is serviced. Conversely, motor warranties focus on covering mechanical failure of major components. Unlike service plans, motor warranties often focus on unscheduled repairing of failed components (Kim et al., 2004; Wang, 2010) with replacement considered if repair is impossible. Table 1 presents examples of service, wear-and-tear and warranty items.

A model to value the unexpired risk of a motor service plan portfolio is important to various stakeholders. Such a model can be used to set provision for motor service plans. Alternatively, the model can be employed to check if provisions are adequate. Moreover, the model can be used to determine economic capital through risk measures such as value at risk on the portfolio run-off at a specific ruin probability. A provider can also utilise the model to project profits emerging from the portfolio's run-off. Taken together, a model to value unexpired risk is important to establish the financial health of a portfolio.

The significance of valuing unexpired risks of motor service plans has increased

over time partly because global trends have seen an increase in warranty period and service plan period offered by the market (Shafiee et al., 2011). Due to increased competitive pressures, providers use longer service plan periods and warranty periods to gain competitive advantage because they signal superior quality and reliability to perspective customers (Murthy & Djamaludin, 2002; Lee et al., 2010). An increase in service plan periods increases the risk taken by providers of these contracts.

Table 1 Examples of Service Items versus Warranty Items

Service Items	Wear-and-tear Items	Warranty/Maintenance Items
Engine oil	Brake pads	Engine
Engine oil filter	Wiper blades	Gearbox
Sump plug seal	Engine drive belt	Differential
Air filter	Brake discs	CV joints
Pollen filter	Rear wheel cylinders	Front suspension
Fuel filter	Shock absorbers	Rear suspension
Spark plugs		Engine control unit
Brake fluid		Transfer box
Clutch fluid		Steering mechanism
Transmission fluid		Emission control unit
Differential fluid		Casings
Timing belt		Radiator
Tensioners		Alternator

Valuing the unexpired risk of a motor service plan portfolio poses practical challenges. One complexity arises from specifying the service plan cover period and servicing intervals in terms of both age (time) and usage (distance travelled). This causes uncertainty on the timing and count of future services. A gap exists in the literature on research focused at designing a reserving model on motor service plans that handles problems stemming from specifying cover and servicing period in two dimensions, particularly from a third party provider's point of view.

The aim of this paper is to contribute towards reducing this gap. To this end, this paper proposes a stochastic discounted cash-flow model to value unexpired risk of a motor service plan portfolio. Cash-flow amount is modelled deterministically. Cash-flow timing is determined by a survival model on time to next service. The

survival model is also employed to determine the probability of an underlying vehicle claiming a scheduled service. For each service plan, the projected count of outstanding future services is obtained though projecting its timing of future services and its corresponding cumulative usage (odometer reading) at future services.

The rest of the paper is organised as follows. Section 2 outlines the problems in reserving for the unexpired risk of motor service plans. In addition, section 2 states the research questions that guide the research. Section 3 presents the model to value unexpired risk. Section 4 presents a numerical example of applying the proposed model based on data from a third party provider. Finally, section 5 concludes.

2. PROBLEM DESCRIPTION AND RESEARCH QUESTIONS

Two main problems in valuing unexpired risk of motor service plans are:

1. The timing and count of future cash-flows is uncertain largely because of the uncertainty in usage rates, meaning distance travelled over a given time. The extant literature has focused mostly on assessing the effect of time and usage on reliability (Murthy & Djamaludin, 2002; Banerjee & Bhattacharjee, 2012) and cost (Kim et al., 2004; Shafiee et al., 2011). In these studies usage rate is often modelled as a constant (Varnosafaderani & Chukova, 2012; Murthy, 1992). Alternatively, usage rate is fitted to a parametric distribution, for example, Gamma distribution (Lawless & Crowder, 2010) and Weibull distribution (Kim et al., 2004). Assuming constant usage rates is inappropriate when projecting future service times from a motor service plan portfolio because usage rates can vary substantially between customers. While parametric distributions account for varied usage rates, they are unable to handle incomplete (censored) usage rate data. Thus, model fitting is done using only complete observations of usage rates. By excluding incomplete usage rate data, parametric modelling results in inefficient use of data readily available to service plan providers. Moreover, time estimates are biased if censored observations are ignored when fitting a time-to-event model (Kaplan & Meier, 1958). To mitigate these problems, this study employs the Kaplan-Meier product limit estimator to model the time taken to cover a service distance. The Kaplan-Meier model handles censored observations and also accounts for varied usage rates. This is achieved without restricting the functional form of the usage rate distribution.
2. To the best of the author's knowledge, the literature lacks an algorithm suitable to simulate unexpired reserve risk of motor service plans. As pointed out by Ohlsson & Lauzeningks (2009), a stochastic approach is often the only way to assess reserve risk in practical situations, unless assuming a normal distribution of the risk is appropriate. This paper describes steps in simulating the unexpired risk reserve for a motor service plan portfolio. The unique aspect of the algorithm is its focus on capturing the risk profile of motor service plans in terms of timing and count of future services.

The following research questions stem from the identified problems:

- How can the time to next service be formulated using a survival model?
- How can partial information on usage be considered when:
  - Estimating the distribution of time to next service?
  - Simulating the timing of the first future service?
- How can a provider estimate the probability of a customer claiming a scheduled service?
- What are the predictive properties of the proposed survival approach?
- How can Monte Carlo simulation be employed to determine provisions of unexpired risk at a specific ruin probability?

3. MODEL

This section develops a model to value unexpired risk of a motor service plan portfolio. Section 3.1 describes the portfolio of motor service plans. In Section 3.2, survival analysis is utilised to model the time to cover a specific distance. Section 3.3 explains how the cost of service is deterministically modelled. Finally, section 3.4 presents the steps to simulate the distribution of the unexpired risk reserve.

3.1 Description of the Portfolio

At a given valuation date, consider a portfolio of N unexpired motor service plan contracts. These contracts, (k=1,…,N), may have different days to expiry, M_k; service distance interval, d_k; and service time interval, t_k. In this study, service distance is measured in kilometres and time is measured in months. For each contract, (d_k, t_k) defines when the underlying vehicle is contractually due for service. For example, if d_k is 15,000 kilometres and t_k is 12 months, it means that the underlying vehicle on contract k is due for service after travelling 15,000 kilometres or after 12 months, whichever occurs first.

3.2 Modelling Time to cover Service Distance Interval

Survival analysis is employed to model time to cover a specific distance. Let T_d represent the time to cover distance d. The distance of interest, in this study, is the service distance interval. T_d is a random variable with survival function:

$$S(t_d) = \Pr(T_d > t_d) \tag{1}$$

In words, the survival function, $S(t_d)$, is the probability that time to cover distance d is longer than a specified time, t_d. The Kaplan-Meier product-limit estimator is used to estimate Equation (1).

Data to estimate Equation (1) is obtained from in-force and expired motor service plans. A complete data record (uncensored) used to estimate Equation (1) is obtained from an individual service plan by evaluating changes in cumulative usage (odometer reading) and time between (1) two consecutive services; and (2) the first

service and service plan start date. Let O_j represent the cumulative usage recorded at the jth service. Further, let τ_j denote the time of the jth service. Assuming a linear usage rate between two consecutive services, the implied time to cover a distance of d is:

$$t_d = \frac{d}{O_{j+1} - O_j}(\tau_{j+1} - \tau_j), \quad j = 0,1,\ldots. \tag{2}$$

Note, $j=0$ represents information at the start of the service plan.

Incomplete data records arise when cumulative usage is unknown on either a service plan's expiry date or the valuation date. To illustrate, consider Figure 1.

The two examples in Figure 1 are from in-force service plans as at the valuation date. Intuitively, the reason why a vehicle has not come for service, as at the valuation date, is that it is not yet due for the next service. This is sensible because a customer has an incentive to return for the next service. This leads to the following proposition:

— Proposition 1 (Distance travelled between last service date and valuation date): For unexpired service plans, distance travelled between the date of last service and the valuation date is less than the vehicle's service distance interval, provided the time period between these dates is less than the service time interval.

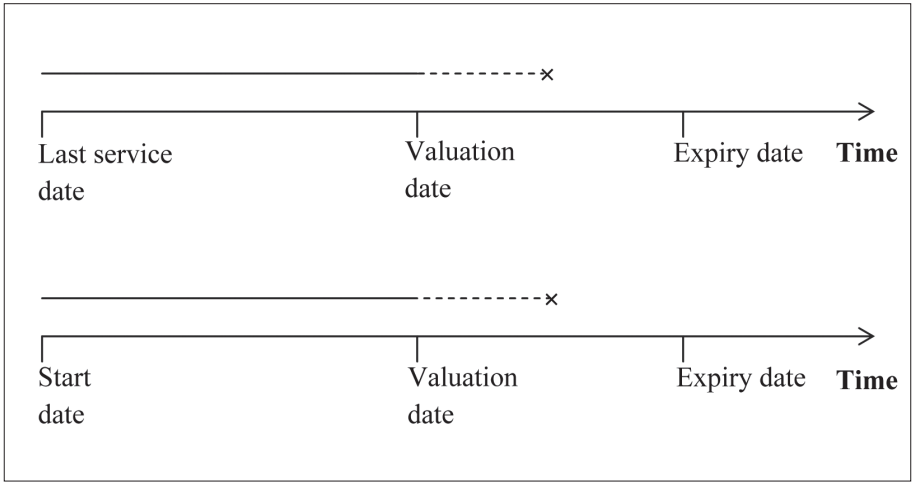

Figure 1 Illustration of Right-censored Data

Note: The continuous straight line, above the time-line, represents the observed part of the time to cover service distance interval, d. In contrast, the dashed line represents the unobserved portion of the time to cover service distance interval, d. The symbol × indicates the time that the service distance interval is reached.

Right-censored data contains partial information on t_d. That is, t_d is greater than the difference between the valuation date and the last service date or service plan start date. The Kaplan-Meier estimator is a classic example of a tool that uses censored data for its partial information.

3.3 Modelling Cost of Service

In this paper, service cost is modelled deterministically. Service cost comprises of the cost of labour, parts and miscellaneous items. Vehicle parts prices can be obtained from a vehicle's manufacturer or other third party data vendors. Servicing dealers are a source of data on labour rates and costing of miscellaneous items.

Alternatively, costing future claims can be based on past service claims, grouped by (1) service type (e.g.: major service; minor service); (2) vehicle make; and (3) vehicle model. This is more reflective of the actual cash outgo to emerge from the service plan portfolio than costing based on service parts pricing obtained from the manufacturer. Past claims experience reflects stringency of the service claims process. Moreover, 'wear-and-tear' items are replaced only if deemed necessary on a given service. For example, if the wear on brake pads is immaterial, then the brake pads might not be replaced, thereby reducing the service cost. In contrast, costing using the parts basket from the manufacturer may include change of brake pads on each service.

3.4 Simulating the Unexpired Risk Reserve of Motor Service Plans

Simulating reserve risk of unexpired motor service plans involves multiple steps. Firstly, the timing of future services is recursively projected for each unexpired service plan. Projecting usage rates utilises past information from (1) the individual service plan; and (2) population usage pattern, meaning the distribution of usage rates from the entire class of business. Partial information on current usage is considered when projecting the first future service. Secondly, the cumulative odometer reading and cost of service are projected for each unexpired risk and projected future service time. Thirdly, the simulated odometer readings and projected future service times of each unexpired risk are used to determine if a provider is on risk. Fourthly, the value of the unexpired risk reserve is calculated as the sum of the discounted value of future service cash-flows from each unexpired service plan, subject to the provider being on risk. An allowance is made that a scheduled service might be unclaimed. These four steps are repeated multiple times resulting in a distribution of the discounted value of the unexpired risk reserve.

The simulation method assumes that:

— Usage rates between service plans are independent and identically distributed. This motivates the use of population usage patterns to infer future usage of an individual service plan.
— The probability of claiming a scheduled service is independent and identically distributed.
— Past usage patterns are a reasonable guide to future usage patterns.

— Future inflation of service costs is constant.
— Discount rate is constant.

The subsections below describe the simulation steps in more detail.

3.4.1 Projecting the Time to Next Service

Let t denote the random variable for time to next service. This is the earlier of (1) t_d, that is, the time to cover service distance d; and (2) t_k, that is, the service time interval for service plan k. Thus:

$$t = \min(t_d, t_k) \tag{3}$$

The service plan contract specifies the value t_k, which is constant for service plan k. In contrast, t_d is a random variable.

To estimate t_d, two data sources are available, namely: (1) data from an individual service plan; and (2) data from a set of expired and in-force service plans. For ease of reference, data from an individual service plan is termed 'own-risk' data, whereas data from a set of expired and in-force service plans is termed 'population' data. The 'population' data is characterised as a survival function of the time to cover a distance d. In contrast, 'own-risk' data is obtained from usage rates observed between services. To illustrate, consider the following example. Say a service plan has had two service claims as at the valuation date. This results in two 'own-risk' usage rate observations as shown in Figure 2. The two usage rate observations shown in Figure 2 can be used to derive 'own-risk' estimates of t_d by using Equation (2). In practice, there are often few 'own-risk' data on motor service plans to perform meaningful survival analysis. This explains why 'own-risk' estimates of t_d exclude right censored observations.

Credibility theory is employed to combine the 'own-risk' and 'population' data when determining the value of t_d. This is desirable because it provides a method to perform experience rating, meaning adjusting future values of t_d based on cumulative past usage experience of an individual service plan. The credibility formula combines the 'own-risk' mean estimate and a value of t_d randomly drawn from the survival

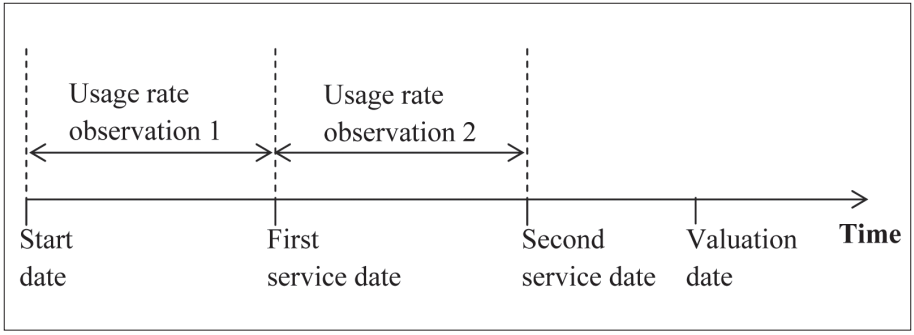

Figure 2 Example of 'Own-risk' Data Observations

function estimated from the 'population' data. Let μ denote t_d that is randomly drawn from a survival function of the 'population' data and $\bar{x}$ denote the 'own-risk' mean estimate of t_d. Let Z denote the weight ascribed to 'own-risk' data. The value of t_d can thus be calculated using a credibility formula as follows:

$$t_d = \bar{x} \times Z + \mu \times (1 - Z), \qquad Z \in (0,1] \tag{4}$$

The proposed formula for Z, a credibility factor, is:

$$Z = \begin{cases} \left(1 - \dfrac{1}{n_k}\right)\left(1 - \dfrac{\sigma}{\bar{x}}\right), \; if\left(n_k \geq \breve{n}\right) and \; (\sigma < \bar{x}) \\ 0, \; Otherwise \end{cases} \tag{5}$$

where

σ = 'own-risk' volatility estimate of t_d.

n_k = count of 'own-risk' observations of t_d. That is, the number of past service claim from service plan k.

$\breve{n}$ = threshold number of 'own-risk' observations deemed as a credible volume of past data.

Three properties of Z are worth noting. First, 'own-risk' data is assigned a non-zero weight if it meets the criteria of having credible volume of past data and low volatility. Volatility is deemed low if $\sigma < \bar{x}$. Second, if the criteria of having credible volume of past data and low volatility are met, then there is a direct relationship between n_k and Z. This can be seen from Equation (6) below:

$$\left.\frac{\partial Z}{\partial n_k}\right|_{n_k \geq \breve{n};\, \sigma < \bar{x}} = \left(\frac{1}{n_k^{\,2}}\right)\left(1 - \frac{\sigma}{\bar{x}}\right) > 0 \tag{6}$$

Third, if the criteria of having credible volume of past data and low volatility are met, then there is an inverse relationship between σ and Z. This can be seen from Equation (7) below:

$$\left.\frac{\partial Z}{\partial \sigma}\right|_{n_k \geq \breve{n};\, \sigma < \bar{x}} = -\left(\frac{1}{\bar{x}}\right)\left(1 - \frac{1}{n_k}\right) < 0 \tag{7}$$

Thus, the properties of Z are characteristic of a credibility factor. That is, Z increases with the sample size of 'own-risk' data and decreases with 'own-risk' volatility (Payandeh Najafabadi, 2010).

A practical drawback to applying credibility theory is that it can considerably increase computational time to simulate the unexpired risk reserve. This drawback is proportionate to the unexpired risk portfolio size. The increase in computational time arises from having to calculate Z for each unexpired risk. Hence if a low computational

time is desired then the decision on whether to apply credibility theory becomes important.

3.4.2 Projecting Time to the First Future Service

Knowing that, as at the value date, a vehicle is yet to claim its next service provides partial detail on usage in the current period. Therefore, there is need to consider this partial information when randomly drawing time to next service, t, from $S(t)$. To illustrate, consider the following example. Say a vehicle without past service history is due for service after 15,000 kilometres or one year, whichever occurs first. Further, say the previous service was three months ago. Intuitively, there is thus need to randomly draw time to next service conditioning on it being greater than three months. To achieve this, one approach is to reject draws where t is at most equal to three months and continue redrawing until the drawn time is greater than three months. Such an approach is however computationally inefficient. A more efficient approach is to ensure that each generated random number results in a simulated value of t that is greater than three months. This can be attained through sampling from a conditional distribution of $S(t)$. Formally, let $\hat{t}$ denote the time from previous service. If a vehicle is yet to have its first service, then $\hat{t}$ denotes time from when the vehicle was sold. The simulated value of t thus satisfies Equation (8).

$$u = S\left(t \mid t > \hat{t}\right) = \frac{S(t)}{S(\hat{t})} \tag{8}$$

In sum, the algorithm to simulate the first future service is:

— Step 1: Estimate $S(t)$.

— Step 2: Calculate $\hat{t}$.

— Step 3: Draw random variate u~Uniform(0,1).

— Step 4: Obtain $S\left(\hat{t}\right)$.

— Step 5: Set t such that $S\left(t\right) = uS(\hat{t})$.

This algorithm closely resembles the Importance Sampling Monte Carlo simulation (ISMCS) approach. In ISMCS, random variates are drawn from a secondary distribution, called the importance distribution, instead of the primary distribution (Lin & Yang, 2011). In this paper, the importance distribution equates to $S\left(\hat{t}\right)$ while the primary distribution is $S(t)$. Desirable properties of ISMCS include: (1) it is a computationally efficient estimator because of its lower variance (Blanchet et al., 2012); and (2) it produces unbiased estimates of the simulated variable (Lin & Yang, 2011).

3.4.3 Projecting Time to Next Services after the First Future Service

For services occurring after the first future service, there is lack of partial information on usage rates. This implies the absence of an importance distribution. Thus, the algorithm to sample from the population usage distribution becomes:

— Step 1: Estimate $S(t)$.
— Step 2: Draw random variate $u \sim \text{Uniform}(0,1)$.
— Step 3: Set t such that $S(t) = u$.

The simulated time to next service can subsequently be used to determine the corresponding dates of future services based on the recursive formula:

$$\tau_{j+1} = \tau_j + t \tag{9}$$

Putting Equation (9) in words: The date of the $(j+1)th$ service is equal to the date of the previous service, τ_j, plus the simulated time to next service, t.

3.4.4 Estimating the Probability of claiming a Scheduled Service

In practice, there is a possibility that a vehicle can forego a scheduled service. This can be due to, for example, the car being involved in an accident. This raises the question: What is the estimated probability of a vehicle foregoing a scheduled service? To answer this question, we can reason as follows. Since a vehicle is at most due for service at the end of its scheduled time interval, it follows that the probability of foregoing a scheduled service is:

$$\Pr\left(\textit{foregoing scheduled service}\right) = S\left(t_k\right) \tag{10}$$

Therefore, the probability of claiming a scheduled service is:

$$\Pr\left(\textit{claiming scheduled service}\right) = 1 - S\left(t_k\right) = F(t_k) \tag{11}$$

In Equation (11), $F(t_k)$ is the inverse of $S(t_k)$, meaning the probability of servicing a vehicle at any time up to the scheduled service time interval.

3.4.5 Determining if a Service Plan is On Risk at a Future Time Period

To establish if a provider is on risk at a future time period, there is need to consider both cumulative usage and time. A dummy variable is employed to indicate whether a motor service plan is on risk at a future date. The value of this dummy variable is determined after two intermediate steps: (1) establishing if the projected cumulative odometer reading at a future date is within a service plan's odometer limit; and (2) determining whether a future date is within a service plan's covered time.

Let

t_{k_j} denote a future time for the jth service on motor service plan k;

$O_{t_{k_j}}$ denote the projected odometer reading at time t_{k_j};

O_k^{LIMIT} denote the odometer limit of service plan k.

Note, t_{k_j} is measured from the valuation date. Dummy variable $I_{t_{k_j}}^{CUMULATIVE\ USAGE}$ indicates if, at time t_{k_j}, motor service plan k has a projected odometer reading within the odometer limit as follows:

$$I_{t_{k_j}}^{CUMULATIVE\ USAGE} = \begin{cases} 1, & O_{t_{k_j}} \leq O_k^{LIMIT} \\ 0, & Otherwise \end{cases} \tag{12}$$

Likewise, dummy variable $I_{t_{k_j}}^{Time}$ indicates if, at time t_{k_j}, motor service plan k is within the covered time period, M_k. That is:

$$I_{t_{k_j}}^{Time} = \begin{cases} 1, & t_{k_j} \leq M_k \\ 0, & Otherwise \end{cases} \tag{13}$$

Given $I_{t_{k_j}}^{CUMULATIVE\ USAGE}$ and $I_{t_{k_j}}^{Time}$, dummy variable $I_{t_{k_j}}^{Time}$ is used to indicate if a provider is on risk at time t_{k_j} as follows:

$$I_{t_{k_j}} = I_{t_{k_j}}^{CUMULATIVE\ USAGE} \times I_{t_{k_j}}^{Time} \tag{14}$$

3.4.6 Projecting the Discounted Value of Future Cost of Services

The value of unexpired risks of a service plan portfolio is obtained by summing the discounted value of projected cost of future services from each motor service plan. An allowance is made for the possibility of customers foregoing a scheduled service. The future services considered are those occurring within the cover period in terms of the service plan's odometer limit and expiry date. Let v_0 denote the value of unexpired risks at time 0. Further, let i denote the effective discount rate; π denote the effective future service cost inflation rate; k denote an index of an unexpired service plan; t_{k_j} denote the time to the jth future service for motor service plan k; M_k denote the days to expiry for the kth motor service plan; $C_{t_{k_j}}$ denote cost of service at time t_{k_j}; $F(t_k)$ denote the probability of a customer claiming a scheduled service; and $I_{t_{k_j}}$ denote an indicator variable that takes a value of 1 if the service at time t_{k_j} is within the service period in terms of both time and cumulative usage. The value of v_0 is thus calculated as:

$$v_0 = \sum_{k=1}^{N} \sum_{t_{k_j} \leq M_k} \frac{C_{t_{k_j}} \left(1+\pi\right)^{t_{k_j}} I_{t_{k_j}} F\left(t_k\right)}{\left(1+i\right)^{t_{k_j}}} \tag{15}$$

Figure 3 illustrates the cash-flow profile of a service plan portfolio. From Figure 3, four points are worth noting. (1) Cash-flows are projected for all N service plans in the portfolio. (2) For each service plan k, cash-flows are projected up to the expiry date, M_k. The time to expiry could vary across the portfolio. For example, in Figure 3,

M_2 is shorter than M_1. (3) The provider may not necessarily be liable for all projected cash-flows occurring before M_k. To illustrate, consider service plan 1 that has three services projected to occur before the service plan's expiry date, M_1. It could be that the projected odometer reading at time t_{1_3} exceeds the odometer limit for the service plan, implying that $I_{t_{1_3}} = 0$. (4) The probability of claiming a scheduled service depends only on the service time interval.

To illustrate, say service plan 2 and service plan N have the same service time interval, that is, $t_2 = t_N$. It follows that the probability of a provider incurring cash-flows $C_{t_{2_1}}$, $C_{t_{N_1}}$ and $C_{t_{N_2}}$ are all equal to $F(t_2)$, which is equal to $F(t_N)$.

3.4.7 Determining Provisions for Unexpired Risks

To set provisions, v_0 is simulated multiple times, thereby producing a distribution of possible outcomes. Let V_0 denote the random variable of the discounted value of unexpired risks of a motor service plan portfolio. From the distribution of V_0, obtained through simulation, risk measures such as value at risk can be computed to set provisions. For example, provisions can be set at, say, the 99% level of confidence. This corresponds to the 99% quantile of V_0's distribution. An alternative risk measure to set provisions is the conditional value at risk (CVaR), which is also known as expected shortfall or expected tail loss (Melnikov & Smirnov, 2012). A desirable property of CVaR is that it is a coherent measure of risk (Artzner et al., 1999).

While this paper focused mostly on evaluating the run-off of existing liabilities, the framework can be adapted to determine provisions over shorter time horizons. For example, the forecasting horizon is limited to one year to evaluate a one-year reserve

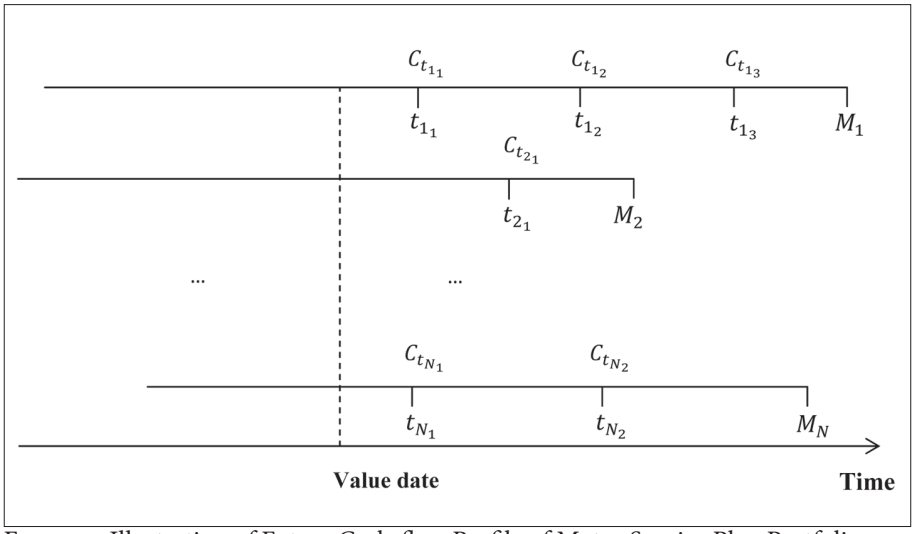

Figure 3 Illustration of Future Cash-flow Profile of Motor Service Plan Portfolio

risk. Globally, such short-term horizon assessment of risk is increasingly becoming popular with regulators of non-life insurance risk (Ohlsson & Lauzeningks, 2009). It is likely that sensitivity of the unexpired risk reserve to usage distribution increases as the risk assessment time horizon reduces. This is because of the increased need for accuracy in timing future services as the modelling time horizon reduces. There is need for due care, however, on placing excessive emphasis on short-term risk measures, particularly if there is a significant portion of extended service plans that come on risk at future dates after the valuation date.

3.5 Back-testing of Model

Goodness of fit testing of the developed model is conducted over different time periods by comparing actual versus expected values on count of services and total claim amount. Back-testing count of services provides insight on the capability of survival models to predict usage patterns. However, since the model aims to quantify the unexpired risk reserve, it is also important to check if the predicted claim amount closely reflects actual emerging claim amounts. Back-testing both count and claim amount is informative of the source of model performance. For example, if count of services is a good fit but total service claims is a bad fit, then this indicates a need to review the costing of future service claims. Expected values in both in-sample and out-of-sample back-testing are obtained through simulating the time to next service. This is then compared to the actual count and claim amounts emerging from the run-off of a fixed portfolio of motor service plans.

Pearson chi-squared test and Mean Absolute Percentage Error (MAPE) is employed to check the predictive accuracy of the count of services and total claim amount. The tests are conducted for both in-sample and out-of sample periods. Data to fit the survival model of usage patterns is derived from the in-sample period, which is also known as the *fit period* (Tashman, 2000). The fitted model is then applied to a separate time period called the out-of-sample period, which is also known as the *test period* (Tashman, 2000). Such out-of-sample testing imitates the situation faced by providers. That is, reserving for unexpired risk is based on inference from a past period. Rossi & Sekhposyan (2011) show that out-of-sample back-testing helps establish if in-sample model performance stems from over-fitting of model. Moreover, out-of-sample back-testing provides insight on stability of the model's predictive capabilities over time (Rapach & Wohar, 2006). Nevertheless, Inoue & Kilian (2004) argue that in-sample back-testing is often more credible than out-of-sample back-testing. Therefore, to optimise on our understanding of model performance, this study uses both in-sample and out-of-sample back-testing.

Regularly back-testing count of services helps to determine when to update the survival curve representing population usage. For example, survival curves estimated from data over consecutive time periods can be tested to establish if the difference between them is statistically significant. Examples of such tests include the log-rank test and Wilcoxon test. The null hypothesis of these tests is that the two survival curves

are equal. Thus, rejecting the null hypothesis is indicative of a need to update the 'population' survival function.

4. NUMERICAL EXAMPLE

4.1 Data

The numerical example is based on sample data from a service plan provider located in South Africa. The service plan portfolio comprised of both manufacturer service plans and extended service plans. The sample motor service plan portfolio was of vehicles meeting the following criteria: (1) on-road passenger vehicles; (2) service distance interval of between 10,000 kilometres and 20,000 kilometres, inclusive; (3) service period interval of one year; and (4) service plans with unexpired risk as at the valuation date, 31 July 2012. Data on service distance intervals and service time interval by vehicle make and model was obtained from a motor data vendor called Diskdrive.[1] Data from Diskdrive covered all vehicle makes and models contained in the service plan provider's portfolio.

Table 2 presents summary statistics on sample data used to estimate the distribution of time to cover a distance of 15,000 kilometres. The *fit period* is from 1 August 2011 to 31 July 2012, while the *test period* is from 1 August 2010 to 31 July 2011. Table 2 shows that censored data can be significant in evaluating usage patterns. This highlights the importance of using survival models in modelling usage patterns since survival models are designed to handle censored observations. The main reason why the sample had a significant portion of censored observations is because it was a relatively young portfolio with few expired policies. Note, 96% of active service plans contributed to the censored observation count.

Table 2 Summary of Data used to estimate Distribution of Time to cover 15,000 km

	Uncensored	Censored	Total
Count	2 560	8 967	11 527
Percentage	22.2%	77.8%	100.0%

Although censored data had the highest proportion of observations, the absolute volume of uncensored data was significant.

4.2 Results

Figure 4 presents the survival curve for the time to cover 15,000 kilometres based on the Kaplan-Meier estimator. Note, 'population' data from the *fit period* is used to derive the survival curve in Figure 4.

Figure 4 shows that it took at least two months for a vehicle with a service distance

1 http://www.diskdrive.co.za

interval of 15,000 kilometres to return for the next service. The 95% confidence bands were small and the survival curve somewhat smooth mostly because of the large sample size. From the survival curve, the probability of claiming a service for a vehicle with a service distance interval of 15,000 kilometres and a service time interval of one year is 97%. This is inferred from $F(12\ month)$, i.e. $1 - S(12\ months)$. Table 3 shows descriptive statistics of the Kaplan-Meier estimator on the time to cover 15,000 kilometres.

Table 3 Kaplan-Meier Estimates of Time (months) to cover 15,000 kilometres

Statistic	Estimate	Confidence Interval (95%)	
		Lower bound	Upper bound
75% quartile	7.2	7.1	7.4
50% quartile	5.3	5.2	5.4
Mean	5.8	5.7	5.9
25% quartile	3.8	3.7	3.9

The mean and median time between consecutive services was five months.

Table 4 presents back-testing results for count of claims. Overall, the model performed poorly when forecasting over a quarterly time horizon based on both the Chi-square test and MAPE. However, there was significant improvement in model performance when using a half-yearly and yearly time period. This is suggestive of a longer model time period to improve model's predictive accuracy.

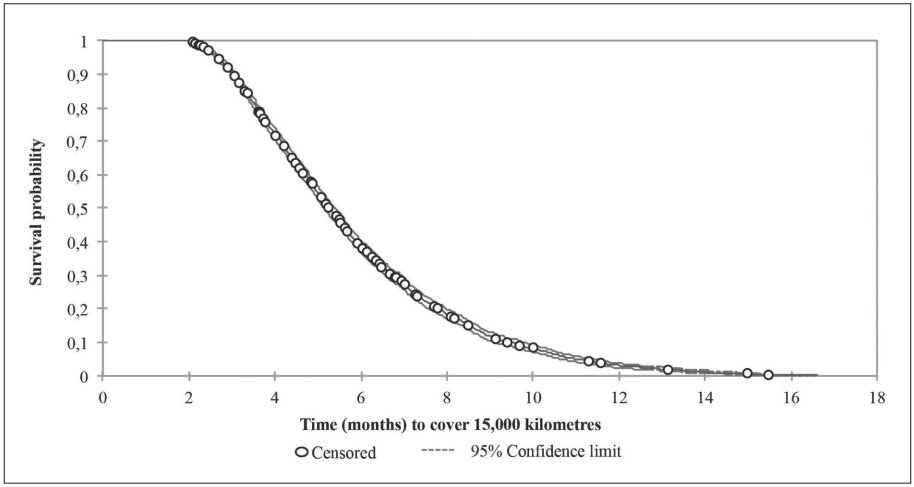

Figure 4 Product-limit Estimate of Time to cover 15,000 kilometres

Table 4 Back-testing Results for Count of Claims

	In-sample				Out-of-sample			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Actual	876	756	845	812	654	524	623	642
Expected	924	804	798	851	700	484	558	651
MAPE								
Quarterly	5.5%				6.6%			
Half-yearly	3.2%				2.5%			
Yearly	2.7%				2.0%			
Chi-squared statistic (p-value)								
Quarterly	9.90 (0.0194)				14.00 (0.0029)			
Half-yearly	5.69 (0.0171)				2.51 (0.1132)			

MAPE denotes Mean Absolute Percentage Error. P-values are in parenthesis.

For longer time periods (yearly and half-yearly), the out-of-sample MAPE was smaller than in-sample MAPE. Although the time horizon to back-test is small, the result of better out-of-sample performance nonetheless suggests stable predictive attributes of the model over time.

4.3 Discussion

Service interval specifications can be materially misleading as an indicator of expected usage from vehicles. For example, the fact that a vehicle is due for service after travelling 15,000 kilometres or after one year, whichever occurs first, does not infer that the average usage rate is 15,000 kilometres per year. It is therefore important to model the distribution of time to next service based on studying past usage patterns to value unexpired risk reserves of motor service plans.

There is need to exercise due care in extrapolating results from the numerical illustration based on a sample portfolio of a single service provider to portfolios of other service plan providers. This is because demographics of service plan portfolios can differ materially across service plan providers. For example, a portfolio covering a small town is likely to have lower usage rates than a portfolio covering a large cosmopolitan area. Further, commercial vehicles are likely to have higher usage rates than personal vehicles.

5. CONCLUSION

This paper developed a stochastic discounted cash-flow model to value the unexpired risk reserve of a motor service plan portfolio. A survival model is used to determine

service cash-flow timing and to estimate the probability that a scheduled service is claimed. Employing a survival model ensures an unbiased estimate of the usage distribution. It also optimises the use of data through utilising partial information on usage. It is shown that taking account of the usage distribution is financially significant in valuing unexpired risk. This is more so if evaluating unexpired risk on a shorter term horizon.

The model in this paper can be further developed in various ways. Examples are the following. First, a survival model that takes account of factors influencing usage could provide insight on the effect of such factors on the value of unexpired reserve risk. A proportional hazard model by Cox (1972) is an example of such a formulation. Second, a study to get more understanding on factors influencing service costs can lead to improved estimates of the value of unexpired reserve risk. Third, a model investigating the likelihood of replacing 'wear-and-tear' components at a particular scheduled service is also a worthwhile research question. Fourth, although this study assumes that usage patterns are time homogeneous, it is conceivable that time-varying usage patterns better capture the risk profile of usage patterns. It is however unclear if the increased complexity of modelling usage patterns as time inhomogeneous would be commensurate with improved model fit. Lastly, the model can be made more dynamic by allowing for the interaction between service claim amount and usage rates.

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APPENDIX

A.1. Notations used in this Paper

Notation	*Description*
N	Number of unexpired motor service plan contracts.
M_k	Days to expiry for the kth motor service plan contract.
d_k	Service distance interval for the kth motor service plan contract.
t_k	Service time interval for the kth motor service plan contract.
i	Effective discount rate.
π	Effective future service cost inflation rate.
t_{k_j}	Time to the jth future service for motor service plan k.
$C_{t_{k_j}}$	Cost of service at time t_{k_j}.
τ_j	Time of the jth service.
O_j	Cumulative usage (odometer reading) at the jth service.
$O_{t_{k_j}}$	Cumulative usage (odometer reading) at time t_{k_j}.
O_k^{LIMIT}	Cumulative usage (odometer reading) limit on service plan k.
t_d	Time to cover distance d.
$\bar{x}$	'Own-risk' mean estimate of t_d.
σ	'Own-risk' volatility estimate of t_d.
n_k	Count of 'own-risk' observations of t_d.
v_0	Value of unexpired reserve risk at time 0.
Z	Credibility factor, meaning weight given to 'own-risk' data.